Nucleon Properties from Approximating Chiral Quark Sigma Model

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Abstract

We apply the approximating chiral quark model. This chiral quark model is based on an effective Lagrangian which the interactions between quarks via sigma and pions-mesons. The field equations have been solved in the mean-field approximation for the hedgehog baryon state. Good results are obtained for nucleon properties in comparison with original model.

Contents

I. Introduction

The description of the processes involving strong interactions is very difficult in the frame of the quantum chromodynamics (QCD) that is due to its non-abelian color and flavor structure and strong coupling constants. These effective models, like linear sigma model, are constructed in such a way as to respect general properties from the more fundamental theory (QCD), such as the chiral symmetry and its spontaneous breaking, e.g. [1, 2]. It is known that the linear sigma model of Gell-Mann and Levy [1] does not always give the correct phenomenology, e.g. the value of pion-nucleon sigma term is too large as in Refs. [3, 4, 5]. Sigma term provides a direct measure of the scalar quark condensates in the nucleon and thereby also constitute an indicator for the mechanism of explicit chiral symmetry breaking. Birse and Banerjee [3] constructed equations of motion treating both σ and $\pi-$ fields as time-independence classical fields and the quarks in hedgehog spinor state that work is reexamined by Broniowski and Banerjee [4]. Birse [5] generalized this mean-filed model to include angular momentum and isospin projections.

Recently, the mesons play a very important role in improving the nucleon properties in the chiral quark models, the perturbative chiral quark model is extended to include the kaon and eta-mesons cloud contributions, to analyze the nucleon properties [6-9]. In the same direction, Horvat et al. [10] applied Tamm-Dancoff method to the chiral quark model which is extended to include additional degrees of freedom as a pseudoscalar-isoscalar field and a triplet of scalar isovector to get better description of nucleon properties. On the other hand, Broniowski and Golli [11] analyzed a particular extension of the linear sigma model coupled to valence quarks, which contained an additional term with gradients of the chiral fields and investigated the dynamically consequence of this term and its relevant to the phenomenology of the soliton models of the nucleon. In same direction, Rashdan et al. [12, 13] considered the higher-order mesonic interactions in the chiral quark sigma model to get a better description of nucleon properties. The aim of this paper is to examine nucleon properties, such as sigma commutator, coupling constant $g_A(0)$ and pion-nucleon coupling constant $g_{\pi NN}(0)$ which are not calculated in Ref. [11].

The paper is organized as follows. Section II: We review briefly approximating chiral quark sigma model. Numerical calculations and the results are presented in section III.

II. APPROXIMATING QUARK SIGMA MODEL

Approximating chiral quark sigma model is described in details in Ref. [11]. The following we give a brief summary.

The Lagrangian density of approximating quark sigma model which describes the interactions between quarks via the $\sigma-$and $\pi-$mesons is written as [11]

$$L\left(r\right)=\overline{\Psi}i\gamma_{\mu}\partial^{\mu}\Psi+\frac{1}{2}\left(\partial_{\mu}\sigma\partial^{\mu}\sigma+\partial_{\mu}\pi.\partial^{\mu}\pi\right)+\frac{1}{2}A_{0}\left(\sigma\partial^{\mu}\sigma+\pi.\partial^{\mu}\pi\right)^{2}+g\overline{\Psi}\left(\sigma+i\gamma_{5}\tau.\pi\right)\Psi$$
$$-U_{1}\left(\sigma,\pi\right), \tag{2-1}$$

with

$$U_{1}\left(\sigma,\pi\right)=\frac{\lambda_{1}^{2}}{4}\left(\sigma^{2}+\pi^{2}-\nu_{1}^{2}\right)^{2}+m_{\pi}^{2}f_{\pi}\sigma, \tag{2-2}$$

is the meson-meson interaction potential where Ψ,σ and π are the quark, sigma and pion fields, respectively. In the mean-field approximation the meson fields are treated as time-independent classical fields. This means that we are replacing powers and products of the

meson fields by corresponding powers and products of their expectation values. The meson-meson interactions in Eq.(2-2) leads to hidden chiral $SU(2) \times SU(2)$ symmetry with $\sigma(r)$ taking on a vacuum expectation value

$$\langle \sigma \rangle = -f_\pi, \tag{2-3}$$

where $f_\pi = 93$ MeV is the pion decay constant. The final term in Eq. (2-2) is included to break the chiral symmetry. It leads to partial conservation of axial-vector isospin current (PCAC). In the original model [3], the A-term is excluded by the requirement of renormalizability. Since we are going to use Eq. (2-1) as an effective model, approximating the underlying quark theory, the model need not and should not be renormalizable as in Refs. [10-13]. The parameters λ_1^2, ν_1^2 are expressed in terms of f_π and the masses $\sigma-$and $\pi-$mesons, we get

$$\lambda_1^2 = \frac{\bar{m}_\sigma^2 - m_\pi^2}{4 f_\pi^2}, \tag{2-4}$$

$$\nu_1^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda_1^2}, \tag{2-5}$$

$$\bar{m}_\sigma^2 = \left(1 + f_\pi^2 A_0\right) m_\sigma^2 \tag{2-6}$$

(for details, see Ref. [11])

Now we expand the extremum, with the shifted field defined as

$$\sigma = \sigma' - f_\pi, \tag{2-7}$$

substituting by Eq. (2-7) into Eq. (2-1), we get

$$\begin{aligned} L(r) = {} & \overline{\Psi} i\gamma_\mu \partial^\mu \Psi + \frac{1}{2}\left(\partial_\mu \sigma' \partial^\mu \sigma' + \partial_\mu \pi . \partial^\mu \pi\right) + \frac{1}{2} A_0 \left(\sigma' \partial^\mu \sigma' + \pi . \partial^\mu \pi\right)^2 - g\overline{\Psi} f_\pi \Psi + \\ & g\overline{\Psi}\sigma'\Psi + ig\overline{\Psi}\gamma_5 . \pi \Psi - U_1(\sigma', \pi) \end{aligned} \tag{2-8}$$

with

$$U_1(\sigma', \pi) = \frac{\lambda_1^2}{4}(\left((\sigma' - f_\pi)^2 + \pi^2 - \nu_1^2\right)^2 + m_\pi^2 f_\pi (\sigma' - f_\pi).$$

The time-independent fields $\sigma'(r)$ and $\pi(r)$ satisfy the Euler-Lagrange equation, and the quark wave function satisfies the Dirac eigenvalue equation. Substituting by Eq. (2-8) in Euler-Lagrange equation as in Ref. [11], we obtain

$$\Box\sigma' = \frac{-1}{(1+((\sigma'-f_\pi)^2+\pi^2)A_0)}\{(1+\pi^2 A_0)((A_0(\sigma'-f_\pi)((\partial^\mu\sigma')^2+(\partial^\mu\pi)^2)+\frac{\partial U_1}{\partial\sigma'}-g\overline{\Psi}\Psi)$$
$$-A_0(\sigma'-f_\pi)\pi((A_0\pi\left((\partial^\mu\sigma')^2+(\partial^\mu\pi)^2\right)+\frac{\partial U_1}{\partial\pi}-ig\overline{\Psi}\gamma_{5}.\tau\Psi)\} \tag{2-9}$$

$$\Box\pi = \frac{-1}{(1+((\sigma'-f_\pi)^2+\pi^2)A_0)}\{(-A_0(\sigma'-f_\pi)\pi)((A_0(\sigma'-f_\pi)((\partial^\mu\sigma')^2+(\partial^\mu\pi)^2)+\frac{\partial U_1}{\partial\sigma'}-$$
$$g\overline{\Psi}\Psi)+(1+(\sigma'-f_\pi)^2A_0)(A_0\pi((\partial^\mu\sigma')^2+(\partial^\mu\pi)^2)+\frac{\partial U_1}{\partial\pi}-ig\overline{\Psi}\gamma_{5}.\tau\Psi)\} \tag{2-10}$$

where τ refers to Pauli isospin matrices and $\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. If $A_0 = 0$, the usual Birse and Banerjee [3] model equations of motion are recovered. Including the color degree of freedom, one has $g\overline{\Psi}\Psi \to N_c g\overline{\Psi}\Psi$ where $N_c = 3$ colors. Thus

$$\Psi(r) = \frac{1}{\sqrt{4\pi}}\begin{bmatrix} u(r) \\ iw(r) \end{bmatrix} \quad \text{and} \quad \bar{\Psi}(r) = \frac{1}{\sqrt{4\pi}}\begin{bmatrix} u(r) & iw(r) \end{bmatrix}, \tag{2-11}$$

and the sigma, pion and vector densities are given by

$$\rho_s = N_c\overline{\Psi}\Psi = \frac{3}{4\pi}\left(u^2-w^2\right), \tag{2-12}$$
$$\rho_p = iN_c\overline{\Psi}\gamma_5\vec{\tau}\Psi = \frac{3}{4\pi}g\left(-2uw\right), \tag{2-13}$$
$$\rho_v = \frac{3}{4\pi}\left(u^2+w^2\right), \tag{2-14}$$

The boundary conditions for the asymptotics for $\sigma(r)$ and $\pi(r)$ at $r \to \infty$ are:

$$\sigma(r) \sim -f_\pi, \qquad \pi(r) \sim 0 \tag{2-15}$$

We used the hedgehog ansatz [3], where

$$\pi(r) = \hat{\mathbf{r}}\pi(r). \tag{2-16}$$

The chiral Dirac equation for the quarks is [12]

$$\frac{du}{dr} = -P(r)u + (W + m_q - S(r))w, \tag{2-17}$$

where the scalar potential $S(r) = g\langle\sigma'\rangle$,the pseudoscalar potential $P(r) = \langle\pi.\hat{r}\rangle$, and W is the eigenvalue of the quarks spinor Ψ.

$$\frac{dw}{dr} = -(W - m_q + S(r))u - \left(\frac{2}{r} - P(r)\right)w. \tag{2-18}$$

III. NUMERICAL CALCULATIONS

A. The scalar field σ'

To solve Eq. (2-9), we integrate a suitable Green's function over the source fields as in Ref. [14]. Thus

$$\sigma'(\mathbf{r}) = \int d^3\mathbf{r}' D_\sigma(\mathbf{r} - \grave{\mathbf{r}})[\frac{-1}{(1+((\sigma'-f_\pi)^2+\pi^2)A_0)}\{(1+\pi^2 A_0)(A_0(\sigma'-f_\pi)((\partial^\mu\sigma')^2 + (\partial^\mu\pi)^2) + \frac{\partial U_1}{\partial\sigma'} - g\rho_s(\grave{\mathbf{r}})) - A_0(\sigma'-f_\pi)\pi(A_0\pi\left((\partial^\mu\sigma')^2 + (\partial^\mu\pi)^2\right) + \frac{\partial U_1}{\partial\pi} - g\rho_p(\grave{\mathbf{r}}))\}] \tag{3-19}$$

where

$$D_\sigma(\mathbf{r} - \grave{\mathbf{r}}) = \frac{1}{4\pi\left|\mathbf{r} - \grave{\mathbf{r}}\right|}\exp(-m_\sigma\left|\mathbf{r} - \grave{\mathbf{r}}\right|),$$

the scalar field is spherical in this model as we only need the $l = 0$ term

$$D_\sigma(\mathbf{r} - \grave{\mathbf{r}}) = \frac{1}{4\pi}\sinh(m_\sigma r_<)\frac{\exp(-m_\sigma r_>)}{r_>}, \tag{3-20}$$

therefore we arrive at the integral equation for $\sigma'(\mathbf{r})$:

$$\sigma'(\mathbf{r}) = m_\sigma\int_0^\infty r'^2 dr'(\frac{\sinh(m_\sigma r_>)}{m_\sigma r_>}\frac{\exp(-m_\sigma r_>)}{m_\sigma r_>})[\frac{-1}{(1+((\sigma'-f_\pi)^2+\pi^2)A_0)}\{(1+\pi^2 A_0)\times \times(A_0(\sigma'-f_\pi)((\partial^\mu\sigma')^2 + (\partial^\mu\pi)^2) + \frac{\partial U_1}{\partial\sigma'} - g\rho_s(\grave{\mathbf{r}})) - A_0(\sigma'-f_\pi)\pi(A_0\pi\left((\partial^\mu\sigma')^2 + (\partial^\mu\pi)^2\right) + \frac{\partial U_1}{\partial\pi} - g\rho_p(\grave{\mathbf{r}}))\}] \tag{3-21}$$

We will solve these implicit integral equation by iterating to self consistency.

B. The pion field π

To solve Eq. (2-10) we integrate a suitable Green's function over the source fields. We use $l = 1$ component of the pion Green's function. Thus

$$\pi(r) = m_\pi\int_0^\infty r'^2 dr'\frac{[-\sinh(m_\pi r_<) + m_\pi r_<\cosh(m_\pi r_<)]}{(m_\pi r_>)^2}\times \times\frac{-1}{(1+((\sigma'-f_\pi)^2+\pi^2)A_0)}\{-A_0(\sigma'-f_\pi)(\pi A_0(\sigma'-f_\pi)((\partial^\mu\sigma')^2 + (\partial^\mu\pi)^2) + \frac{\partial U_1}{\partial\sigma'} - g\rho_s(\grave{\mathbf{r}})) + (1+(\sigma'-f_\pi)^2 A_0)(A_0\pi((\partial^\mu\sigma')^2 + (\partial^\mu\pi)^2) + \frac{\partial U_1}{\partial\pi} - g\rho_p(\grave{\mathbf{r}})))\} \tag{3-22}$$

We have solved Dirac Eqs. (2-17, 2-18) using fourth order Rung-Kutta method. Due to the implicit nonlinearly of these Eqs. (2-9, 2-10) it is necessary to iterate the solution until self-consistency is achieved. To start this iteration process we use the chiral circle form for the meson fields:

$$S(r) = m_q(1-\cos\theta) \text{ and } P(r) = -m_q \sin\theta, \tag{3-23}$$

where $\theta = \tanh r$

C. Properties of the Nucleon

The proton and neutron magnetic moments are given by

$$\mu_{p,n} =< P\uparrow \left| \int d^3\mathbf{r}\frac{1}{2}\mathbf{r}\times \mathbf{j}_{\varepsilon M}(\mathbf{r}) \right| P\uparrow>, \tag{3-24}$$

where, the electromagnetic current is

$$j_{\epsilon M}(\mathbf{r}) = \bar{\Psi}\left(\mathbf{r}\right)\gamma\left(\frac{1}{6}+\frac{\tau_3}{2}\right)\Psi(\mathbf{r}) - \varepsilon_{\alpha\beta_3}\pi_\alpha\left(\mathbf{r}\right)\nabla\pi_\beta\left(\mathbf{r}\right), \tag{3-25}$$

such that

$$\left(\mathbf{j}_{\epsilon M}(\mathbf{r})\right)_{nucleon} = \bar{\Psi}\left(\mathbf{r}\right)\gamma\left(\frac{1}{6}+\frac{\tau_3}{2}\right)\Psi\left(\mathbf{r}\right), \tag{3-26}$$

$$\left(\mathbf{j}_{\epsilon M}(\mathbf{r})\right)_{meson} = -\epsilon_{\alpha\beta 3}\pi_\alpha\left(\mathbf{r}\right)\nabla\pi_\beta\left(\mathbf{r}\right), \tag{3-27}$$

The nucleon axial-vector coupling constant is found from

$$\frac{1}{2}g_A(0) = \left\langle P\uparrow \left| \int d^3r A_3^z(\mathbf{r}) \right| P\uparrow \right\rangle, \tag{3-28}$$

where the z-component of the axial vector current is given by

$$A_3^z(\mathbf{r}) = \bar{\Psi}\left(\mathbf{r}\right)\frac{1}{2}\gamma_5\gamma^3\tau_3\Psi\left(\mathbf{r}\right) - \sigma\left(\mathbf{r}\right)\frac{\partial}{\partial z}\pi_3\left(\mathbf{r}\right) + \pi_3\left(\mathbf{r}\right)\frac{\partial}{\partial z}\sigma\left(\mathbf{r}\right). \tag{3-29}$$

The pion-nucleus σ commutator is defined

$$\sigma(\pi N) = \left\langle P\uparrow \left| \int d^3r\sigma'(\mathbf{r}) \right| P\uparrow \right\rangle, \tag{3-30}$$

In calculation of $\sigma(\pi N)$, we replace $\sigma'(\mathbf{r})$ by $\frac{j_\sigma(\mathbf{r})}{m_\sigma^2}$ where $j_\sigma(\mathbf{r})$ is the source current defined by

$$(\Box + m_\sigma^2)\sigma' = j_\sigma(\mathbf{r})$$

To calculate the pion-nucleon coupling constant, we consider the limit $\mathbf{q} \longrightarrow 0$ of

$$\frac{g_{\pi NN}(0)}{2M} =< P \uparrow \left| \int d^3\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \times \mathbf{j}_{\pi 3}(\mathbf{r}) \right| P \uparrow >, \tag{3-31}$$

where pion source current is defined by

$$(\Box + m_\pi^2) = \mathbf{j}_{\pi 3}(\mathbf{r}) \tag{3-32}$$

(For details see Refs. [3-5]).

D. Discussion of Results

The field equations (2.9$\rightarrow$ 2.18) have been solved by iteration as in Ref. [10] for different values of quark and sigma masses. Tables 1, 2 and 3 show the nucleon observables calculated for m_q =400 - 480 MeV and m_σ = 441 MeV and m_σ = 900 MeV, respectively.

From Table I, the nucleon properties are calculated at m_σ = 441 MeV which is predicted by Chiral Perturbation Theory (ChPT) [15]. Note, magnetic moments of the nucleon are improved by increasing the quark mass that backs to the increases of mesonic interactions with quarks. Also, Sigma commutator $\sigma(\pi N)$ is improved by decreasing the quark mass. The similar situation is hold for pion-nucleon coupling constant $(g_{\pi NN}(0)\frac{m_\pi}{2M_N})$ which improved by decreasing the quark mass, $g_A(0)$ is not little sensitive for changing in quark mass which backs to the expression of axial vector current (Eq. 3-29) is not depend on explicitly of quark mass but only on the dynamic of fields.

From Table II, the nucleon properties are calculated at m_σ = 900 MeV as in Ref [11]. We obtain good results at m_q =462 MeV which is consistent with NJL model as in Ref. [16]. From Tables (I, II), we note the $g_A(0)$ is little sensitive for sigma mass that backs to the same reason as pointed out before. Also, the properties of the nucleon are not sensitive for sigma mass [11], so we take two extreme values m_σ = 441MeV and m_σ = 900 MeV which is compatible with Refs. [11, 15].

We known sigma commutator is important quantity to measure the breaking chiral symmetry and it is one of problems in the full chiral sigma model as in Refs. [3, 4, 5]. Recently, Sigma commutator is predicted by perturbative chiral quark model [7] and lattice QCD [17]

which the value obtained lies in the range 45 to 55 MeV. The effect A-term is strongly on this quantity that the change in range 30% relative to full model [3]. Good value obtained equal 56 MeV at ($m_q = 400$ and $m_\sigma = 900$)MeV (see, Table II)

From Fig. 1, we see the sigma field passes through zero at $r = 0.5$, which we will refer to as the soliton radius ($r = 0.5$), whereas at $r \to \infty$ the pion field $\to 0$ and sigma field $\to -f_\pi$. The pion field takes the shape of the P-wave, which gives the attraction of the pion-quark interaction, and goes to zero in a linear manner for large distances. We also see that the meson fields do not stray far from the circular minimum of the potential $\sigma^2 + \pi^2 = f_\pi^2$. The pion field reaches its maximum value close to the soliton radius. Fig. II. shows the components of quarks $u(r)$ and $w(r)$ corresponding the fields.

Table I. Values of magnetic moments of the nucleon, $\sigma(\pi N)$ term, $g_A(0)$ and $g_{\pi NN}(0)\frac{m_\pi}{2M_N}$. At $m_6 = 441$ MeV. $f_\pi^2 A_0 = -0.02$. All quantities in MeV.

m_q (MeV)	400	420	440	462	480
$\mu_p\,(N)$	2.572	2.640	2.699	2.757	2.845
$\mu_n\,(N)$	-1.873	-1.951	-2.018	-2.085	-2.135
$\sigma\,(\pi N)$	69	76	81	84	85
$g_A(0)$	1.689	1.714	1.734	1.752	1.764
$g_{\pi NN}(0)\frac{m_\pi}{2M_N}$	1.315	1.365	1.410	1.454	1.485

Table II. Values of magnetic moments of the nucleon, $\sigma(\pi N)$ term, $g_A(0)$ and $g_{\pi NN}(0)\frac{m_\pi}{2M_N}$. At $m_6 = 900$ MeV., $f_\pi^2 A_0 = -0.03$. All quantities in MeV.

m_q (MeV)	400	420	440	462	480
$\mu_p\,(N)$	2.509	2.581	2.632	2.676	2.706
$\mu_n\,(N)$	-1.908	-1.983	-2.039	-2.087	-2.120
$\sigma\,(\pi N)$	56	62	66	69	71
$g_A(0)$	1.733	1.757	1.774	1.787	1.794
$g_{\pi NN}(0)\frac{m_\pi}{2M_N}$	1.27	1.318	1.354	1.388	1.417

IV. COMPARISON WITH OTHER MODELS

It is interesting to compare the nucleon properties in the present approach with other models. Here we consider two models: Perturbative Chiral Quark Model [6-9] and Original

sigma model [3]. The perturbative chiral quark model is an effective model of baryons based on chiral symmetry. The baryon is described as a state of three localized relativistic quarks supplemented by a pseudoscalar meson cloud as dictated by chiral symmetry requirements. In this model the effect of the meson cloud is evaluated perturbatively in a systematic fashion. The model has been successfully applied to the nucleon properties (see Table III). We obtained reasonable results in comparison with this model which backs to perturbative chiral quark model based on non-linear $\sigma-$ model Lagrangian and leads to good description of nucleon properties (for details, see Refs. [6-9]). In particular, nucleon magnetic moments are improved in comparison with this model. In comparison with full model of Birse and Banerjee [3]. We note the most observables are sensitive for A-term that best results are obtained at ($m_q = 462$ and $m_\sigma = 900$)MeV

Table III. Observables of the nucleon comparison with Quark Sigma Model [3] and Perturbative Chiral Quark Model [6-9]

Quantity	$m_\sigma = 900$ MeV $m_q = 462$ MeV	[3]	[6-9]	Exp.
$\mu_p\,(N)$	2.68	2.87	2.62±0.02	2.79
$\mu_n\,(N)$	-2.08	–2.29	-2.0±0.02	-1.91
$\sigma(\pi N)$	69	92	54.7	-
$g_A(0)$	1.78	1.86	1.19	1.25
$g_{\pi NN}(0)\frac{m_\pi}{2M_N}$	1.38	1.53	-	1.0

Fig. 1: Sigma and pion fields (in units of f_π) as functions in distance R for $m_\sigma = 900$ MeV, $m_q = 462$ MeV, $f_\pi^2 A_0 = -0.03$

Fig. 2: The components of quark (in units of f_π) as function in distance R for $m_\sigma = 900$ MeV, $m_q = 462$ MeV, $f_\pi^2 A_0 = -0.03$

V. CONCLUSION

From the results, the most nucleon properties are improved in comparison with original model. In particular, sigma commutator is improved in range 30% in comparison with full model. $g_A(0)$ is little sensitive for A-term, so we need to test the quantum effects or increase mesonic interactions in this approach to improve this quantity in future works.

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VII. Re fernces

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